QUIZ 14 SOLUTIONS: LESSON 20
OCTOBER 18, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

Let

$$f(x, y) = \frac{x - y}{1 - x}.$$

1. [2 pts] Find f_x.

   Solution: We will need to use either the quotient rule or the product rule. Here, I will use the product rule because that should make further derivatives easier to handle. To use the product rule I need to do a quick rewrite:

$$\frac{x - y}{1 - x} = (x - y)(1 - x)^{-1}.$$

   We write

$$\begin{aligned} f_x(x, y) &= \frac{\partial}{\partial x}\left((x - y)(1 - x)^{-1}\right) \\ &= (x - y)\left[\frac{\partial}{\partial x}(1 - x)^{-1}\right) + (1 - x)^{-1} \underbrace{\left[\frac{\partial}{\partial x}(x - y)\right]}_{1} \\ &= (x - y)\left[(-1)(-1)(1 - x)^{-2}\right] + (1 - x)^{-1} \\ &= \boxed{(x - y)(1 - x)^{-2} + (1 - x)^{-1}} \\ &\text{OR } \boxed{\frac{1 - y}{(1 - x)^2}} \end{aligned}$$

2. [2 pts] Find f_y.

   Solution: This derivative is more straight forward:

$$f_y(x, y) = \frac{\partial}{\partial y}\left(\frac{x - y}{1 - x}\right) = \frac{1}{1 - x}\left[\frac{\partial}{\partial y}(x - y)\right] = \frac{1}{1 - x}(-1) = \boxed{-\frac{1}{1 - x}}.$$

3. [2 pts] Find f_{xx}.

   Solution: Again, I'll be using the product rule here.

$$\begin{aligned}
f_{xx}(x,y) &= \frac{\partial}{\partial x}(f_x) \\
&= \frac{\partial}{\partial x}\left((x-y)(1-x)^{-2} + (1-x)^{-1}\right) \\
&= \frac{\partial}{\partial x}\left((x-y)(1-x)^{-2}\right) + \frac{\partial}{\partial x}((1-x)^{-1}) \\
&= (x-y)\left[\frac{\partial}{\partial x}((1-x)^{-2})\right] + (1-x)^{-2}\left[\frac{\partial}{\partial x}(x-y)\right] + (-1)(-1)(1-x)^{-2} \\
&= (x-y)\left[(-2)(-1)(1-x)^{-3}\right] + (1-x)^{-2} + (1-x)^{-2} \\
&= \boxed{2(x-y)(1-x)^{-3} + 2(1-x)^{-2}} \\
&\text{OR } \boxed{\frac{2(1-y)}{(1-x)^3}}
\end{aligned}$$

4. [2 pts] Find f_{yy}.

Solution:

$$f_{yy}(x,y) = \frac{\partial}{\partial y}\left(-\frac{1}{1-x}\right) = \boxed{0}$$

5. [2 pts] Find f_{xy}.

Solution:

$$\begin{aligned}
f_{xy}(x,y) &= \frac{\partial}{\partial y}(f_x) \\
&= \frac{\partial}{\partial y}\left[(x-y)(1-x)^{-2} + (1-x)^{-1}\right] \\
&= \frac{\partial}{\partial y}((x-y)(1-x)^{-2}) + \underbrace{\frac{\partial}{\partial y}((1-x)^{-1})}_{0} \\
&= (1-x)^{-2}\frac{\partial}{\partial y}(x-y) \\
&= \boxed{-(1-x)^{-2}}
\end{aligned}$$